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2021-2022 EVEN

ANNEXURE I

CLASS WISE SUBJECT DETAILS

DEPARTMENT OF MATHEMATICS

S.NO	CLASS	NAME OF THE SUBJECT
1	II B Sc MATHS	TRIGONOMTRY & ANALYTICAL GEOMETRY OF 3D
2	II B Sc MATHS	DYNAMICS
3	II B Sc MATHS	INFERENTIAL STATISTICS
4	III B Sc MATHS	MODERN ALGEBRA-II
5	III B Sc MATHS	REAL ANALYSIS-II
6	III B Sc MATHS	COMPLEX ANALYSIS-II
7	III B Sc MATHS	NUMERICAL ANALYSIS
8	III B Sc MATHS	GRAPH THEORY

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ANNEXURE- II

WILLINGNESS REPORT

From

Dr.T.Ragunathan, M.Sc., M Phil., Ph.D

Asst. Professor in Mathematics,

Department of Mathematics,

PGP college of Arts and science,

Namakkal-.

To

The Principal,

PGP college of Arts and Science,

Namakkal.

Sir/ Madam,

Sub: Willingness Report for Subject: - Reg.

I hereby express my willingness to handle the following subject in the following order of priority.

Order of Priority	Name of the Subject	Class	Reason for Selection
1	Real Analysis-II	III B.Sc Maths	Interseted
2	Inferential Statistics	II B.Sc Maths	Interseted

Thanking You

Date: 28-02-2022

Time:

Copy to: 1. HOD of Mathematics

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Signature of the Staff

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ANNEXURE III

MINUTES OF SUBJECT ALLOCATION MEETING

I Dr.T.Ragunathan hereby submit the minutes of the department meeting held for subject allotment on 02.03.2022 at 11 am based on annexure I &II

S.No	Name of the Staff	Name of the Subject	Class	No of Hours	Signature of the Staff
1.	Dr.T.Ragunathan	Real Analysis-II	III B.Sc Maths	6	[signature]
		Inferential Statistics	II B.Sc Maths	5	[signature]

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CIRCULAR

DATE: 03-02-2022

Intimation of course allotment for faculties in the Department of Mathematics during EVEN SEMESTER/ 2021-2022 for BSc students

S.No	Name of the Faculty	Title of the Course	Course code	Class & Year
1.	Dr.T.Ragunathan	Real Analysis-II		III BSc
		Inferential Statistics		II BSc

Note: Faculties are asked to follow the syllabus issued by Periyar University, Salem.

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Copy to

1. The HOD
2. All faculties of Mathematics
3. File copy

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TIME TABLE

EVEN SEM (2021-2022)

NAME OF THE FACULTY: Dr.T.Ragunathan

S.NO	CLASS	PAPER NAME	HOURS
1	III BSc	Real Analysis-II	6
2	II BSc	Inferential Statistics	5
		TOTAL	**11**

Day/ Hours	I	II	III	IV	V	VI
I	II B Sc M	III B Sc M			III B Sc M	
II				II B Sc M		
III	III B Sc M					II B Sc M
IV	II B Sc M	III B Sc M				
V	III B Sc M		II B Sc M			
VI						

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UNIT-3

PARTICULARS	PROPOSED DATE	COMPLETED DATE	TEACHING AIDS	REFERENCE
1.Test Hypothesis	13.04.2022	13.04.2022	PPT	Gupta. S. C. And Kapoor. V. K
2.Statistical Hypothesis	18.04.2022	19.04.2022	PPT	"
3.Types of Errors	20.04.2022	21.04.2022	PPT	"
4. Neyman-Pearson Lemma.	22.04.2022	22.04.2022	PPT	"

UNIT-4

PARTICULARS	PROPOSED DATE	COMPLETED DATE	TEACHING AIDS	REFERENCE
1.Sampling Distribution	04.05.2022	04.05.2022	PPT	Gupta. S. C. And Kapoor. V. K
2.Standard Errors	07.05.2022	07.05.2022	PPT	"
3. Large Samples	09.05.2022	10.05.2022	PPT	"
4 . Simple Problems	12.05.2022	13.05.2022	PPT	"

UNIT-5

PARTICULARS	PROPOSED DATE	COMPLETED DATE	TEACHING AIDS	REFERENCE
1.Test of significance.	18.05.2022	20.05.2022	PPT	Gupta. S. C. And Kapoor. V. K
2.t and F test	21.05.2022	23.05.2022	PPT	"
3.chi square test	24.05.2022	26.05.2022	PPT	"
4. Good ness of fit test	27.05.2022	31.05.2022	PPT	"

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UNIT PLAN

EVEN SEM (2021-22)

NAME OF THE FACULTY: Dr.T.RAGUNATHAN

SUBJECT: **CLASS:** II B Sc MATHS

SUBJECT CODE: 19USTA01 **NO. OF HOURS / WEEK:** 05

THEORY BASED SYLLABUS

UNIT-1

PARTICULARS	PROPOSED DATE	COMPLETED DATE	TEACHING AIDS	REFERENCE
1.Population and sample	17.03.2022	18.03.2022	PPT	Gupta. S. C. And Kapoor. V. K
2. consistency	18.03.2022	19.03.2022	PPT	,,
3. Unbiasedness	22.03.2022	22.03.2022	PPT	,,
4. Efficiency	24.03.2022	25.03.2022	PPT	,,

UNIT-2

PARTICULARS	PROPOSED DATE	COMPLETED DATE	TEACHING AIDS	REFERENCE
1.Methods of estimations.	28.03.2022	29.03.2022	PPT	Gupta. S. C. And Kapoor. V. K
2.Properties of estimators	02.04.2022	03.04.2022	PPT	,,
3.Maximum likehood,Methods of moments	06.04.2022	06.04.2022	PPT	,,
4. Interval estimators .	09.04.2022	09.04.2022	PPT	,,

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STUDY MATERIAL
EVEN SEM (2021-22)

NAME OF THE FACULTY: Mr.T.RAGUNATHAN

ALLIED STATISTICS

FOR B.Sc., (MATHS) / B.Sc., (MATHS) C.A. P. Code: 19USTA03

(For the candidates admitted from 2017 – 2018 onwards)

SEMESTER II or IV: Allied I or II Paper II: Inferential Statistics

Unit – I: Population and Sample; Parameter and Statistic – Point and Interval Estimation – Consistency, Unbiasedness, Efficiency (Cramer – Rao Inequality) and Sufficiency (Rao – Blackwell Theorem)

Unit – II: Method of Estimation – Maximum Likelihood and Methods of Moments – properties of these estimators – Interval Estimation (Concept only)

Unit – III: Testing of Hypothesis – Concept of Statistical Hypothesis – Simple and Composite hypothesis – Null and alternative Hypothesis – Critical Region – Type I and Type II Errors – Power of a Test – Neyman-Pearson Lemma.

Unit – IV: Test of Significance – Sampling distribution – Standard Error – Large Sample Tests with regard to Mean, Difference of Means, Proportions and Difference of Proportions – Simple Problems.

Unit – V: Test of Significance – Exact sample test based on t and F distributions with regard to Means, Variance and Correlation Coefficient – Chi-square tests for single Variance, Goodness of fit and Independence of attributes.

Reference Books:

Gupta. S. C. And Kapoor. V. K (2004) – Fundamentals of Mathematical Statistics – (11th edition), Sultan Chand & Sons, New Delhi.

Sancheti. D. C. And Kapoor. V. K, Statistics (7th Edition), Sultan Chand & Sons, New Delhi.

NAME OF THE PAPER: INFERENTIAL STATISTICS

...ample:

μ : population mean

σ^2: variance population variance

Finite population:

A population may include countable number of observations is called finite population

ex Students in a college

Infinite population

A population may include uncountable number of observations is called infinite population.

ex:

Water in the sea

Estimator:

The part statistics measures obtained from the sample observation is called estimation.

Ex:

$\bar{x}$ - sample mean

S^2 - sample variance

Estimate:

The particular value of an estimator (T_n) is called an estimate of the population.

Ex:

Sample mean ($\bar{x}$) is an estimate of population mean (μ)

...umeter space:

The set of all possi...

Unit - III

Testing of hypothesis

Statistics hypothesis:-

A statistical hypothesis is some statement about population (or) equivalently about the probability distribution, characteristic of a population which we want to verify on the basics of the information available from the sample.

Type of hypothesis:-

Parametric hypothesis

Non-parametric hypothesis

parametric hypothesis

The statistical constants of unknown values of population is called parametric

Ex:

Mean (μ), variance (σ^2) and standard deviation (σ) is the population parametre.

Statistics:-

The statistical measures computed

Level of signification

The probability of α that a random value of the Statistic belongs to the critical region is no action level of significance is the sizes of the type I error.

The level of significance is the used in 1%, 5% and 10%.

Degree of Freedom (D.O.F)

Based on the problem to fixed the degree of freedom.

Table value.

Based on the degree of freedom on the level of Significance identify the value from the Statitical value.

Conclusion:

Since calculated value (cv) is less than table value are accept the null hypothesis. Since calculated value is greater than table value are rejected the null hypothesis.

Type I and Type II error. [error in

Procedure of testing of hypothesis:

we know sumorise below the various testing of a statistical hypothesis in a systematic manner.

Step:1

Identify the problem to the applied in the consor manner.

Step:2 (Null hypothesis : H_0)

The technic of randomisation used for selection of a sample unit makes the test of signification valued for as such a hypothesis which is has a hypothesis has no difference is a Null hypothesis. It is denoted by H_0.

The Null hypothesis is the hypothesis which is tested for possible rejection under the assumption that is true IT is denoted by H_0

Alternation hypothesis:

Any hypothesis which is complemen-torry to null hypothesis is called Alternati...

2m Composite hypothesis with 'r' degrees of Freedom.

A Statistical hypothesis which does not Specify completly 'r' parametres of a population is called a composite hypothesis with 'r' degrees of Freedom.

Sample Space:-

Let $x_1, x_2 \ldots x_n$ be the sample observations denoted by 'C' all the values are of 'a' will be aggreegate of a sample and they constitude a space is called the Sample space. which is denoted by 'S'.

2m Critical region:

The region of rejection of Ho when Ho is true is that the region of out come set, when Ho is rejected it the sample point fals in region is called critical region.

Power of the test:

$1-\beta$ is the power function of

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If $p(x \in w/H_0) = \int_w L_0 \, dx \quad \alpha = 1$

And, $p(x \in w/H_1) \geq p(x \in w_1/H_1) \; \forall \theta \cdot \theta_0$ — (2)

For every other critical region w_1 satisfying equ (1)

10m Neymann pearson Lemma

Statement:

Let $k > 0$ be a constant and w be a critical region of size α such that $w = \left\{ x \in S : \frac{f(x, \theta_1)}{f(x, \theta_0)} > k \right\}$

$$w = \left\{ x \in S : \frac{L_1}{L_0} > k \right\}$$

and $$\bar{w} = \left\{ k \in S : \frac{L_1}{L_0} < k \right\}$$

where,

L_0 and L_1 are the likelihood fuction of the sample observation $(x = x_1, x_2 \ldots x_n)$ and under H_0 and H_1 respectively the w is the most powerful critical region of the test hypothesis $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$

Proof:

Type - II error:-

Accept H_0, when H_0 is wrong

$P(\text{Accept } H_0/H_1) = \beta$

	True	False
Rejected H_0	type I error α	Correct
Accept H_0	Correct	Type II error

2m

Simple and Composite hypothesis:-

i) In the statistical hypothesis Specify the population complotly then its is called a simple hypothesis.

Ex:

Consider a population $N(\mu, \sigma^2)$

The hypothesis $H_0 : \mu = \mu_0$

ii) If the statistical hypothesis does not specify the population composite then it is called a composite hypothesis

Ex:

Show that we have $P(x \in w_{1}/H_0)$

$$= \int_{W_1} L_0 \, dx = \alpha_1 \quad (3)$$

$$P(x \in W_1/H_1) = \int_{W_1} L_1 \, dx = 1-\beta_1 \quad \text{———} \quad (4)$$

Now we have to prove that

$$1-B \geq 1-B_1$$

Let $W = A \cup C$ and

$W_1 = B \cup C$ (C may be empty W_0 and W_1 is may be disjoined

If $\alpha_1 \leq \alpha$

$$\int_{W_1} L_0 \, dx \leq \int_{W} L_0 \, dx$$

$$W_1 = B \cup C = B + C$$

$$W = A \cup C = A + C$$

$$\int_{B} L_0 \, dx + \int_{C} L_0 \, dx \leq \int_{A} L_0 \, dx + \int_{C} L_0 \, dx$$

$$\int_{B} L_0 \, dx \leq \int_{A} L_0 \, dx$$

$$\int_{A} L_0 \, dx \geq \int_{B} L_0 \, dx \quad \text{———} \quad (5)$$

We have ...

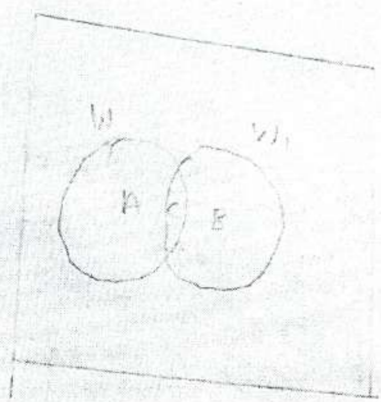
Show that we have $P(x \in w_1 / H_0)$

$$= \int_{w_1} L_0\, dx = \alpha_1 \quad \text{------} \quad (3)$$

$$P(x \in w_1 / H_1) = \int_{w_1} L_1\, dx = 1 - \beta_1 \quad \text{------} \quad (4)$$

Now we have to prove that

$$1 - \beta \geq 1 - \beta_1$$



Let $w = A \cup C$ and
$w_1 = B \cup C$ (C may be empty w_0 and w_1 is may be disjoined

If $\alpha_1 \leq \alpha$

$$\int_{w_1} L_0\, dx \leq \int_{w} L_0\, dx$$

$$w_1 = B \cup C = B + C$$

$$w = A \cup C = A + C$$

$$\int_B L_0\, dx + \int_C L_0\, dx \leq \int_A L_0\, dx + \int_C L_0\, dx$$

$$\int_B L_0\, dx \leq \int_A L_0\, dx$$

$$\int_A L_0\, dx \geq \int_B L_0\, dx \quad \text{------} \quad (5)$$

$$\Rightarrow \int_A L_1 dx > k \int_A L_0 dx \geq k \int_B L_0 dx \quad \text{--- (6)}$$

Also we have $\frac{L_1}{L_0} \leq k$ in $\bar{\omega}$ (or) ω_1

$$L_1 \leq k L_0 \text{ in } \omega_1$$

Since, B contains ω_1

$$\int_B L_1 dx \leq k \int_B L_0 dx$$

$$\Rightarrow \int_B L_1 dx \leq k \int_B L_0 dx \leq \int_A L_0 dx$$

$$\int_B L_1 dx \leq \int_A L_0 dx$$

which implise

$$\int_B L_1 dx \leq k \int_B L_0 dx \qquad \int_B L_1 dx \leq k \int_B L_0 dx \leq k \int_A L_0 dx$$

$$\int_B L_1 dx \leq \int_A L_1 dx$$

$1-\beta_1 \leq 1-\beta$

$1-\beta \geq 1-\beta_1$

H.T.P

$$\int_B L_1 dx + \int_C L_1 dx \leq \int_A L_1 dx + \int_C L_1 dx$$

bothsides $\int_C L_1 dx$ addin

$$\int_{B \cup C} L_1 dx \leq \int_{A \cup C} L_1 dx$$

$$\int_{\omega_1} L_1 dx \leq \int_{\omega_0} L_1 dx$$

Unbiased test and unbiased critical region:-

Let us consider the testing of $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ the critical region ω and consequanty

Let p be the probability that a coin will fall head in a single toss in order to test $H_0: p = 1/2$ and $H_1: p = 3/4$. The coin is tossed 5 times or H_0 is rejected if more than 3 heads are obtain. Find the probability of type errors and power of the test.

Given that, $H_0 : p = 1/2$ and $H_1 : p = 3/4$

If the random variable x denotes the number of heads in n tosses of a coin

$X \sim B(n, p)$

$$P(X=x) = nC_x p^x q^{n-x}$$

$$\therefore n = 5$$

$$P(X=5) = 5C_x p^x q^{5-x}$$

The critical region is,

$$w = \{x : x \geq 4\} \text{ and}$$

$$\bar{w} = \{x : x \leq 3\}$$

α = probability of type-I errors

$$= P[x \in w / H_0]$$

$$= P[x \geq 4 / H_0 : p = 1/2]$$

$$= P[x \geq 4 / H_0 : p = 1/2]$$

$$= P[x = 4 / H_0 ; p = 1/2] + P[x = 5/4 : p = 1/2]$$

$$= 5C_4 (1/2)^4 (1/2)^1 + 5C_5 (1/2)^5 (1/2)^0$$

$= P[x \in \omega / H_1]$

$= P[x \leq 3 / H_1 : p = 3/4]$

$= 1 - P[x \geq 3/H_1 : p = 3/4]$

$= 1 - P[x \geq \frac{3}{H_1} : p = 3/4]$

$= 1 - \{P(x = 4/H_1 : p = 3/4)\} + P[x = 5/H_1 : p = 3/4]$

$= 1 - \{5C_4 (3/4)^4 (1/4)^1 + 5C_5 (3/4)^5 (1/4)^{10}\}$

$= 1 - \frac{81}{128}$

$= \frac{47}{128}$

Power of the test $= 1 - \beta$

$= \frac{1 - 47}{128} = \frac{81}{128}$

If $x \geq 1$ is the critical region $H_0 : \theta = 2$ against the alternative $\theta = 1$. And the basic of the single observation from the population $f(x, \theta) = \theta \exp(-\theta x)$, $0 \leq x \leq \infty$ obtain the value of type I and type II errors.

Sol:

Given that $H_0 : \theta = 2$, $H_1 : \theta = 1$

Critical region ω = [...

$$= \int_1^{\infty} 2 e^{-2x} \, dx$$

$$= 2\int_1^{\infty} e^{-2x} \, dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_1^{\infty}$$

$$= -\left[e^{-\infty} - e^{-2}\right]$$

$$= e^{-2}$$

$$\alpha = 1/e^2$$

β = Probability of type-II error

$$= P(x \in \bar{w} / H_1)$$

$$= P[x < 1 / H_1 : \theta = 1]$$

$$= \int_0^1 f(x, \theta) \, dx$$

$$= \int_0^1 1 \cdot e^{-x} \, dx$$

$$= \int_0^1 e^{-x} \, dx$$

$$= \left[\frac{e^{-x}}{-1}\right]_0^1$$

$$= -\left[e^{-1} - e^{0}\right]$$

$$= 1 - e^{-1}$$

Given the Frequency of $f(x,\theta)$ a

$F(x,\theta) = \begin{cases} 1/\theta, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$ and the

your testing null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$ by mean single of observed value of x. What would be the size of type-I and type-II error. If you choose for the interval

i) $0.5 \le x$ ii) $1 \le x \le 1.5$ as the critical region also obtain the power of test.

Sol:

i) $0.5 \le x$, $H_0: \theta = 1$, $H_1: \theta = 2$

Critical region $w = \{x \ge 0.5\}$

$\bar{w} = \{x : x < 0.5\}$

α = Probability of Type I error

$= P[x \in w / H_0]$

$= P[x \ge 0.5 / H_0 : \theta = 1]$

$= \int_{0.5}^{1} f(x,\theta)\, dx$

$= \int_{0.5}^{1} \frac{1}{1}\, dx$

$= (x]_{0.5}^{1}$

β = probability of type II error

$= P[x \in \bar{\omega} / H_1]$

$= P[x < 0.5 / H_1 : \theta = 2]$

$= \int_{0}^{0.5} f(x, \theta)\, dx$

$= \int_{0}^{0.5} 1/2\, dx$

$= 1/2 \int_{0}^{0.5} dx$

$= 1/2 [x]_{0}^{0.5}$

$= 1/2 [0.5 - 0]$

$= \frac{1}{2}(0.5)$

$\beta = 0.25$

power of the test $= 1 - \beta$

$= 1 - 0.25$

$= 0.75$

$\leq x < 1.5$

Let x follows $N(\mu, 4)$, μ is unknown to test $H_0 : \mu = -1$ against $H_1 : \mu = 1$ based on a sample of size 10 from this population we gives the critical region $x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10} \geq 0$ what is the power of the test.

Sol:

Given $X \sim N(\mu, 4)$

$H_0 : \mu = -1$

$w = \{x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10} \geq 0\}$

Let $u = x_1 + 2x_2 + 3x_3 + \cdots + 10x_{10}$

Since x_i's are identically and

Unit - IV

Test of Significance [large sample] $n > 30$

Definition:

The very important aspects of Sample theorey. is the study of the test of significane which enable us to deside on the basic of the sample result.

If the deviation between the observed sample statistics and the hypothetical parameter value or the deviation between the two independent sample statistics is significant.

2m
Standard error:-

The standard deviation of the sample distribution of a statistic is known as it standard error.

A standard error of sum of well known statistic for the large sample are given below

α = Probability of type I error

$= P[x \in \omega / H_0]$

$= P[u \geq 0 / H_0]$ —— ①

under $H_0 : \mu = -1$ $u \sim N[-55, 1540]$

$$z = \frac{u - E(u)}{\sqrt{V(u)}} = \frac{u - (-55)}{\sqrt{1540}}$$

$$= \frac{u + 55}{39.2428}$$

When $\mu = 0$

$$z = \frac{55}{39.2428}$$

$$z = 1.4015$$

$\alpha = P[u \geq 0 / H_0] = P[z \geq 1.40]$

$= 0.5 - P[0 \leq z \leq 1.40]$

$= 0.5 - 0.4192$

$\boxed{\alpha = 0.083}$

Power of the test in given by

$1 - \beta = P[x \in \omega / H_1]$

When $u = 0$

$$z = \frac{-55}{39.2428}$$

$= -1.4015$

$1-\beta = P[u \geq 0/H_0] = P[z \geq 1.40]$

$= 0.5 + P[0 \leq z \leq 1.40]$

$= 0.5 + 0.4192$

$1-\beta = 0.9192$

$\beta = -0.9192 + 1$

$\boxed{\beta = 0.0808}$

Given the Frequency function $F(x, \theta) = \frac{1}{\theta}, 0 \leq x \leq \theta$ and that your testing the hypothesis $H_0: \theta = 1.5$ against $H_1: \theta = 2.5$ by means of single observed value of x. what would be the size of Type-I and Type-II error, If you succes choose that the interval $0.8 \leq x$ as the critical region also obtain the power of test.

Sol:

α = Probability of Type-I error

$= P[x \in \omega / H_0]$

$= \int_{0.8}^{1.5} f(x,\theta)\, dx$

$= \int_{0.8}^{1.5} \frac{1}{\theta}\, dx$

$= \int_{0.8}^{1.5} \frac{1}{1.5}\, dx$

$= \frac{1}{1.5} [x]_{0.8}^{1.5}$

$= \frac{1}{1.5} (1.5 - 0.8)$

$= \frac{0.7}{1.5}$

$\alpha = 0.4667$

β = Probability of Type-II error

$= \int_{0}^{0.8} f(x,\theta)\, dx$

$= \int_{0}^{0.8} \frac{1}{\theta}\, dx$

$= \int_{0}^{0.8} \frac{1}{2.5}\, dx$

Let p denote the probability of getting a heads when a coin is tossed once. Suppose that the hypothesis $H_0 : \theta = 0.5$ is rejected in of $H_1 : \theta = 0.6$. If 10 trails result in 7 or more heads calculate the probability of Type-I and Type-II error.

The critical region is

$$w = \{x : x \geq 7\}$$

α = Probability of type-I error.

$$\alpha = P[x \in w / H_0]$$

$$= P[x \in 7 / H_0]$$

$$= P[x \in 7 / H_0 : P = 0.5]$$

Here $n = 10$, $p = 0.5$, $q = 0.5$

$$= P[x = 7/H_0 : P = 0.5] + P[x = 8/H_0 : P = 0.5] + P[x = 9/H_0 : P = 0.5] + P[x = 10/H_0 : P = 0.5]$$

$$= 10C_7 (0.5)^7 (0.5)^3 + 10C_8 (0.5)^8 (0.5)^2 + 10C_9 (0.5)^9 (0.5)^1 + 10C_{10} (0.5)^{10} (0.5)^0$$

$$= (120 + 45 + 10 + 1)(0.5)^{10}$$

$$= (176)(0.0009765)$$

$$\alpha = 0.17186$$

β = Probability of Type-II error

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$= 1 - P[x \geq 7/H_1 ; P = 0.6]$

Here $n = 10$, $p = 0.6$, $q = 0.4$

$= 1 - P[x = 7/H_1 ; P = 0.6] + 1 - P[x = 8/H_1 ; P = 0.6] + 1 - P[x = 9/H_1 ; P = 0.6] + 1 - P[x = 10/H_1 ; P = 0.6]$.

$= 1 - [10C_7 (0.6)^7 (0.4)^3 + 10C_8 (0.6)^8 (0.4)^2 + 10C_9 (0.6)^9 (0.4) + 10C_{10} (0.6)^{10} (0.4)^0]$

$= 1 - [(120 + 45 + 10 + 1)(0.6)^{10}]$

$= 1 - [120(0.02799)(0.064) + 45(0.01679)(0.16) + 10(0.01007)(0.4) + 1(0.0060)]$

$= 1 - [0.21496 + 0.12088 + 0.04 + 0.006]$

$= 1 - [0.38184]$

$\beta = 0.6181$

6. Difference between of two sample mean } $(\bar{x}_1 - \bar{x}_2)$ $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

7) Difference of two sample S.D (σ) } $(S_1 - S_2)$ $\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}$

8) Difference of two Sample Proportion } $(P_1 - P_2)$ $\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}$

Table value for large sample test level of significance

Critical value	1%	5%	10%
two tailed test	2.58	1.96	1.645
Right tailed (>)	2.33	1.645	1.28
Left tailed (<)	-2.33	-1.645	-1.28

S.m ①
S.D - ①

1) Test of significance for single mean
2) Test of significance for Diffrence b/w two mean → S.m ②, SD - ②, P.S.P(or) ...
3) Test of significance for single proportion
4) Test of significance for Difference between proportion.

1) Test of ...

(or)

Standard deviation is known as,

$$|z| = \frac{\bar{x} - \mu}{S/\sqrt{n}} \sim N(0,1)$$

$\bar{x}$ - sample mean
μ - population mean
σ - population standard deviation
S - Sample standard deviation
n - number of observation

$$S = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2}$$

$$S = \sqrt{\left(\frac{\Sigma f x^2}{\Sigma f}\right) - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

$$S = \sqrt{\frac{\Sigma f m^2}{\Sigma f} - \left(\frac{\Sigma f m}{\Sigma f}\right)^2}$$

(X) 5m

1) If sample of 900 members has mean of 3.4cm and standard deviation 2.61cm if the sample from a large population of a mean 3.25 cm standard deviation 2.61 standard deviation if ...

Null hypothesis H_0:

There is no significance difference b/w Sample mean and population mean.

Alternative hypothesis H_1:

There is significance difference b/w Sample mean and population mean.

Test Statistic:

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{3.4 - 3.25}{2.61/\sqrt{900}}$$

$$= \frac{0.15\,(30)}{2.61}$$

$$Z = 1.724$$

Level of significance:

At 5% Level of significance

Table value

5% Level of significance table value is 1.96.

Conclusion:

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DEPARTMENT OF MATHEMATICS

Continuous Assessment Test I

CLASS : II B.Sc Maths **INFERENTIAL STATISTICS** **Date : 06.05.2022 FN**

Time : Two hours **Maximum : 50 marks**

PART A — (10 × 1 = 10 marks) Answer ALL questions.

1. A measure characterizing a sample such as x or s is called

(a) population (b) statistic (c) universe (d) mean

2. Sample mean is an unbiased estimator ————.

(a) population mean (b) sample mean (c) both (a) and (b) (d) all the above

3. A point estimation is a —————.

(a) lies between numbers (b) three numbers (c) single number (d) all the above

4. Interval estimation is a —————.

(a) single number (b) lies between two numbers (c) both (a) and (b) (d) all the

5. In maximum Likelihood estimator, $\partial^2 L / \partial\theta^2$ —————.

(a) <0 (b) >0 (c) =0 (d) all the above

6. $\mu r'$ = —————.

(a) $\sum_{i=1}^{n} x_i^r$ (b) $\frac{1}{n}\sum_{i=1}^{n} x_i^r$ (c)) $\frac{1}{2n^2}\sum_{i=1}^{n} x_i^{2r}$ (d) $\sum_{i=1}^{n} x_i$

7. Write the difference between statistic and parameter.

8. A statistic is:

a) a sample characteristic b) a population characteristic c. unknown d. normally distributed

9.Which of the following symbols represents a population parameter?

a) SD b) σ c) r d) 0

10. As a general rule, researchers tend to use __ __ percent confidence intervals.

a) 99% b) 95% c) 50% d) none of the above

PART B — (2 × 5 = 10 marks) Answer any TWO questions

11. Define : (a) consistency estimator (b) unbiased estimator.

12. State the properties of maximum likelihood estimator.

13.Write short notes on Interval Estimation.

PART C — ($3 \times 10 = 30$ marks) Answer Any Three questions.

14. State and prove Cramer-Rao inequality

15. State and prove Rao-Blackwell theorem.

16. Find the MLE of the parameter λ , when X follows Poisson distribution.

17. Derive confidence interval for difference of two proportions.

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Manju

INFERENTIAL STATISTICS

* Raw Blackwell Theorem
* Crammer Row Inequality

C. MANJU
II - B.SC MATHEMATICS

Function $h(y/x)$

$$h(y/x) = \frac{F(x,y)}{F(x)}$$

$$h(y/x) = F(x,y)$$

$$F(x,y) = h(y/x)\, F(x) \rightarrow (1)$$

$$E\left[\frac{y}{X=x}\right] = \int_{-\infty}^{\infty} y \cdot h(y/x)\, dy$$

$$= \int_{-\infty}^{\infty} y, \frac{f(x,y)}{f(x)}\, dy$$

$$= \frac{1}{f(x)} \int_{-\infty}^{\infty} y F(x,y)\, dy$$

$$\phi(x) = \frac{1}{F(x)} \int_{-\infty}^{\infty} y F(x,y)\, dy$$

Since the conditional distribution $E\left(\frac{y}{x=x}\right)$ is independent of parameter μ.

Hence x is a sufficient estimator of μ.

$$E[\phi(x)] = E\left[E\left(\frac{y}{X=x}\right)\right]$$

$$= E(y).$$

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$$h(y/x) = \frac{F(x,y)}{F(x)}$$

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$$E[\phi(x)] = E\left[E\left(\frac{y}{X=x}\right)\right]$$

$$= E(y)$$

$= E\left[y - \phi(x)\right]^2 + E\left[\phi(x) - E(\phi(x))\right]^2$

$V(y) = E\left[y - \phi(x)\right]^2 + V\left[\phi(x)\right]$

$V\left[\phi(x)\right] = V(y) - E\left[y - \phi(x)\right]^2$

$V\left[\phi(x)\right] \leq V(y)$

Hence the proof..

Crammer Row Inequality.

Statement:

If t is an unbaised estimator of $\gamma(\theta)$. A function of the parameter θ, Then.

$$V(t) \geq \left[\frac{d\gamma(\theta)}{d\theta}\right]^2 \Big/ E\left[\frac{\partial \log L}{\partial \theta}\right]^2$$

$$\geq \gamma'(\theta)^2 / I(\theta).$$

$$= E\left[y-\phi(x)\right]^2 + E\left[\phi(x) - E(\phi(x))\right]^2$$

$$V(y) = E\left[y-\phi(x)\right]^2 + V\left[\phi(x)\right].$$

$$V\left[\phi(x)\right] = V(y) - E\left[y-\phi(x)\right]^2$$

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